BIBLIOGRAPHICAL NOTE AND QUERY.

BY MARCUS BAKER, U. S. COAST SURVEY, WASHINGTON, D. C.

IN *The Mathematical Diary* etc., conducted by *James Ryan*, A. M., 12°, New York, 1832, volume II, No. 13, p. 311, we find the following:—

"QUESTION XXVI. (269.) or PRIZE QUESTION.—*By Scientific Sigma, Esq., New-York.*

"It is required to describe upon the same plane, three circles touching each other, each of which shall touch two given circles.

"For the best solution to this qustion, a handsomely bound complete set of the Diary in two volumes, is offered."

Now this number of the Diary, March 1832, was, according to Dr. Hart, the last that appeared. See ANALYST, Vol. II, p. 134. And since it was the last no solution could have appeared in Ryan's Diary. Can any reader of the ANALYST tell whether it has ever been taken up since then and solved, and if so, when, where and by whom?

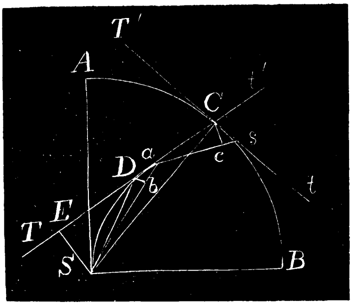
It may be added that, before it was here proposed as the Prize Question, it had been solved by Prof. Jacob Steiner of Berlin, whose solution may be found in Crelle's Journal, Vol. I, pp. 180—181. No proof of the solution was given by Steiner and, so far as I am aware, none has ever been given.

PROBLEM. BY W. E. HEAL, WHEELING, IND. — "I have a circular fish pond, radius, r; a duck is swimming around the edge and my dog starts from the center in pursuit of the duck, swimming always directly towards it. Required the equation, and length, of the curve described by the dog, supposing that he swims n times as fast as the duck.

[The foregoing is substantially the same question which was published, together with the following solution by us, in the *School Friend* for Aug., 1851. Though our solution is here submitted, if any of our readers should feel sufficient interest in the question to obtain a different, or more elegant, solution, we will be pleased to insert it in a future number.—Ed.]

Solution. Let S represent the center of the pond and ACB one fourth of its circumference, and let D denote any position of the dog and C the corresponding position of the duck; then must curve $SD = n \times \text{arc } AC$.

Draw Tt tangent to the curve at D and $T't'$ tangent to the circle at C. Draw Sa and Ds indefinitely near SD and DC, respectively, and draw Db and Ce respectively perpendicular to Sa and Ds.



Join SC. Then is SC perpendicular to $T't'$, and because, at the limit, $\angle DCe$ is a right angle, therefore, because $\angle SCe$ is common to the two right angles, DCe and SCs, $\angle SCE = \angle sCe$. And because the angles SEC and seC are right angles, therefore $\angle ESC = \angle esC$ and the triangles CES and Ces are similar.

Put $SE = x$ and $SD = y$, then is $DE = \sqrt{(y^2-x^2)}$. Also, put curve $SD = nu$, then will $Da = ndu$ and $Cs = du$, and by the similar triangles ECS and eCs we have

$$r : x :: du : es = \frac{xdu}{r}.$$

Also, (Eucl. 47, I.) $EC = \sqrt{(r^2-x^2)}$, and, consequently,

$$DC = \sqrt{(r^2-x^2)} - \sqrt{(y^2-x^2)}, \tag{1}$$

the differential of which is

$$\frac{-xdx}{\sqrt{(r^2-x^2)}} - \frac{ydy-xdx}{\sqrt{(y^2-x^2)}}.$$

Now, while the dog moves over the space Da the duck moves from C to s and therefore the line DC is increased by the distance es and diminished by the distance Da. Hence the differential of the line DC is $-Da+es$, or

$$-ndu+\frac{xdu}{r} = \left(\frac{x-nr}{r}\right)du = \text{differential of } DC.$$

Equating this value of the differential of DC with that found from (1), we have

$$\left(\frac{x-nr}{r}\right)du = \frac{-xdx}{\sqrt{(r^2-x^2)}} - \frac{ydy-xdx}{\sqrt{(y^2-x^2)}};$$

or

$$du = \frac{-rxdx}{(x-nr)\sqrt{(r^2-x^2)}} - \frac{rydy-rxdx}{(x-nr)\sqrt{(y^2-x^2)}}. \tag{2}$$

Again, by the similar triangles Dab and SDE we have

$$ndu : dy :: y : \sqrt{(y^2-x^2)},$$

whence

$$du = \frac{ydy}{n\sqrt{(y^2-x^2)}}. \tag{3}$$

Equating the right-hand members of (2) and (3) we have

$$\frac{-rxdx}{(x-nr)\sqrt{(r^2-x^2)}} - \frac{rydy-rxdx}{(x-nr)\sqrt{(y^2-x^2)}} = \frac{ydy}{n\sqrt{(y^2-x^2)}},$$

whence

$$\frac{-nrdx}{\sqrt{(r^2-x^2)}} = \frac{nrdx+ydy}{\sqrt{(y^2-x^2)}}. \tag{4}$$

Equation (4) is the differential equation to the curve sought; but, as the second member is not an exact differential, it is doubtful whether the equation can be integrated in finite terms.

If any of our readers will effect an integration of (4), by series or otherwise, so as to obtain the value of x^2 in functions of y suitable for substitution in (3), and will calculate the numerical value of u for any given values of n and r we will insert the discussion, in detail, in No. 6 of the ANALYST.

THE POLYNOMIAL THEOREM.

BY MISS CHRISTINE LADD, POQUONOCK, CONN.

FOR the expansion of the nth power of m quantities the ordinary Algebras offer no better way than to apply the Binomial Theorem m—1 times. The process is laborious and the result is not easily brought into a symmetrical form.

Let us write down the sum of m quantities n times, and see how the actual multiplication is performed:

$$\begin{array}{l} a+b+c+d+\ldots. \\ a+b+c+d+\ldots. \\ a+b+c+d+\ldots. \\ \ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots \end{array}$$

It is evident that the terms of the product will consist of every possible combination of one or more of the quantities with different exponents in such a manner that the sum of the exponents is equal to n. The only difficulty is to determine the coefficients.

The coefficient of a term expresses the number of ways in which that term can be formed. a^n, b^n &c. can be formed in only one way, therefore tbeir coefficient is 1. The term $a^{n-1}b$ is formed by taking the b of any row and multiplying it by the a of every other row, but as any one of the n b's can be taken there are n terms $a^{n-1}b$, or the coefficient of $a^{n-1}b$ is n; and all terms of the same form, $b^{n-1}d$, $a^{n-1}c, \ldots$ have the same coefficient. To obtain the coefficient of $a^{n-2}b^2$ we observe that the number of ways in which two b's can be selected out of n b's is $\frac{1}{2}n(n-1)$. Take the form $a^{n-5}b^3c^2$. We can take the two c's in $\frac{1}{2}n(n-1)$ ways. Having taken a certain combination of 2 c's we have $n-2$ b's out of which to select 3 b's. This can be done in $\dfrac{(n-2)(n-3)\ldots(n-2-3+1)}{3!}$ ways, and only one